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## Moments of inertia for solids of revolution and variational methods

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### Abstract

We present some formulae for the moments of inertia of homogeneous solids of revolution in terms of the functions that generate the solids. The development of these expressions exploits the cylindrical symmetry of these objects, and avoids the explicit use of multiple integration, providing an easy and pedagogical approach. The explicit use of the functions that generate the solid gives the possibility of writing the moment of inertia as a functional, which in turn allows us to utilize the calculus of variations to obtain a new insight into some properties of this fundamental quantity. In particular, minimization of moments of inertia under certain restrictions is possible by using variational methods.

**Keywords:** Moment of inertia, variational methods, solids of revolution.  
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The moment of inertia (MI) is a very important concept in Physics and Engineering [1]. In this paper, we present simple formulae to obtain the MI's of homogeneous solids of revolution. The expressions presented here are written in terms of the functions that generate the solid and only require simple integration. Finally, we show that minimization of the moment of inertia under certain restrictions is possible by employing the calculus of variations.

### 1 MI of solids of revolution generated around the $X$ -axis

Figure 1a shows a function that generates a solid of revolution around the  $X$ -axis (we shall call it the "generating function" henceforth). The narrow rectangle of height  $f(x)$  and width  $dx$  generates a thin disk of height  $dx$  and radius  $f(x)$ . We shall calculate the moment of inertia of the solid of revolution generated by  $f(x)$  with respect to the axis of symmetry ( $X$ -axis).

We know from the literature [1], that the MI of a thin disk

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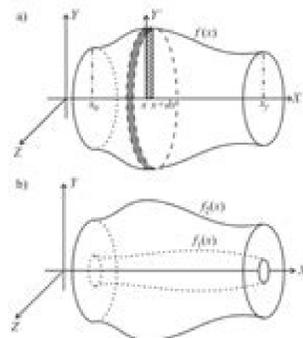


Figure 1: Solid of revolution generated from the  $X$ -axis. (a) With one generating function  $f(x)$ . (b) With two generating functions  $f_1(x)$ ,  $f_2(x)$ .

with respect to the  $X$ -axis is given by  $(1/2)MR^2$  where  $M$  is the mass of the disk and  $R$  gives its radius. Thus, the differential MI for our thin disk reads

$$dI_X = \frac{1}{2}(dM)f(x)^2, \quad (1)$$

The differential of mass is given by

$$dM = \rho dV = \rho \pi f(x)^2 dx, \quad (2)$$

where  $\rho$  denotes the density of the solid, and will be assumed constant throughout the document. Substituting Eq. (2), into Eq. (1) and integrating we get

$$I_X = \frac{\pi\rho}{2} \int_{x_0}^{x_1} f(x)^4 dx. \quad (3)$$

Eq. (3) gives the MI of any solid of revolution with respect to the axis of symmetry. Now let us calculate the MI with respect to the  $Y$ -axis. To do it, we first estimate the MI of the thin shaded disk around the axis  $Y'$  shown in Fig. 1a. It is well known that for an axis passing through the diameter

### Experimental Mass Moment of Inertia Part A

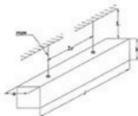
#### Equipment

Model LS-2108, a stopwatch, a weighing scale, a measuring tape, and 3 lengths of steel wire.

#### Objectives:

1. The experiment is to determine experimentally the moment of inertia and radius of gyration of a rectangular bar.
2. Comparison between experimental and theoretical results.

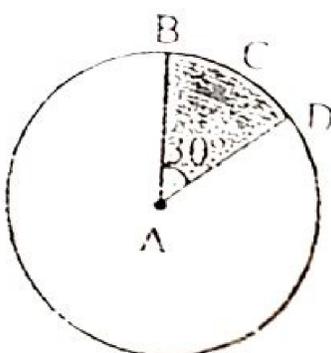
#### Experimental procedure:



1. The values of length of rope,  $L$ , centre distance,  $r$ , mass of rectangular bar,  $M$ , and the dimensions of bar  $a$ ,  $b$ , and  $l$  are obtained and measured.
2. The bar is connected with eye bolts with negligible mass onto the wires with the clips provided.
3. The bar is twisted horizontally to put it into oscillation about its vertical axis to a simple harmonic motion.
4. The bar is left until it oscillates in a steady state.
5. The time is obtained in seconds taken for 20 oscillations.
6. The value is divided with the number of oscillations to obtain time taken for 1 oscillation. The experiment is repeated to obtain time taken for 30, 40, 50, and 60 oscillations to obtain a more precise value for the period in seconds.
7. The value of the radius of gyration,  $K$ , and moment of inertia,  $I$  are calculated using the following equation provided.

$$t = 2\pi \sqrt{\frac{I}{Mg r^2}} \quad \text{or} \quad t = 2\pi \sqrt{\frac{K^2 L}{r^2 g}} \quad \text{for} \quad I = MK^2$$

The moment of inertia of a uniform flat disc about its own axis is  $I$ . The radius of the disc is a  $A$  section ABCD of the disc (as shown in figure) is cut off and separated. The moment of inertia of the remaining part of the disc about the same axis will be





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